Module 7

Transformer

Lesson 23

Ideal Transformer

Contents

23.1 Goals of the lesson

In this lesson, we shall study two winding *ideal transformer*, its properties and working principle under no load condition as well as under load condition. Induced voltages in primary and secondary are obtained, clearly identifying the factors on which they depend upon. The ratio between the primary and secondary voltages are shown to depend on ratio of turns of the two windings. At the end, how to draw phasor diagram under no load and load conditions, are explained. Importance of studying such a transformer will be highlighted. At the end, several objective type and numerical problems have been given for solving.

Key Words: Magnetising current, HV & LV windings, no load phasor diagram, reflected current, equivalent circuit.

After going through this section students will be able to understand the following.

1. necessity of transformers in power system.
2. properties of an ideal transformer.
3. meaning of load and no load operation.
4. basic working principle of operation under no load condition.
5. no load operation and phasor diagram under no load.
6. the factors on which the primary and secondary induced voltages depend.
7. fundamental relations between primary and secondary voltages.
8. the factors on which peak flux in the core depend.
9. the factors which decides the magnitude of the *magnetizing current*.
10. What does loading of a transformer means?
11. What is reflected current and when does it flow in the primary?
12. Why does VA (or kVA) remain same on both the sides?
13. What impedance does the supply see when a given impedance Z_2 is connected across the secondary?
14. Equivalent circuit of ideal transformer referred to different sides.

23.2 Introduction

Transformers are one of the most important components of any power system. It basically changes the level of voltages from one value to the other at constant frequency. Being a static machine the efficiency of a transformer could be as high as 99%.

Big generating stations are located at hundreds or more km away from the load center (where the power will be actually consumed). Long transmission lines carry the power to the load centre from the generating stations. Generator is a rotating machines and the level of voltage at which it generates power is limited to several kilo volts only –

a typical value is 11 kV. To transmit large amount of power (several thousands of mega watts) at this voltage level means large amount of current has to flow through the transmission lines. The cross sectional area of the conductor of the lines accordingly should be large. Hence cost involved in transmitting a given amount of power rises many folds. Not only that, the transmission lines has their own resistances. This huge amount of current will cause tremendous amount of *power loss* or I^2r loss in the lines. This loss will simply heat the lines and becomes a *wasteful energy*. In other words, *efficiency of transmission* becomes poor and cost involved is high.

The above problems may addressed if we could transmit power at a very high voltage say, at 200 kV or 400 kV or even higher at 800 kV. But as pointed out earlier, a generator is incapable of generating voltage at these level due to its own practical limitation. The solution to this problem is to use an appropriate *step-up transformer* at the generating station to bring the transmission voltage level at the desired value as depicted in figure 23.1 where for simplicity single phase system is shown to understand the basic idea. Obviously when power reaches the load centre, one has to step down the voltage to suitable and safe values by using transformers. Thus transformers are an integral part in any modern power system. Transformers are located in places called *substations*. In cities or towns you must have noticed transformers are installed on poles – these are called *pole mounted distribution transformers*. These type of transformers change voltage level typically from 3-phase, 6 kV to 3-phase 440 V line to line.

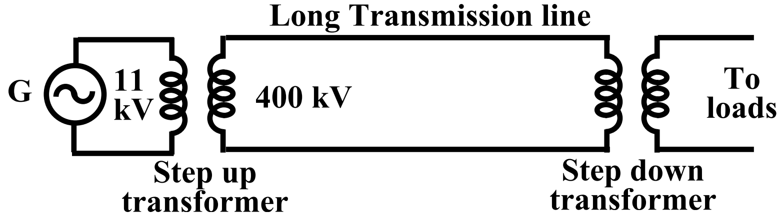


Figure 23.1: A simple single phase power system.

In this and the following lessons we shall study the basic principle of operation and performance evaluation based on equivalent circuit.

23.2.1 Principle of operation

A transformer in its simplest form will consist of a rectangular *laminated magnetic structure* on which two coils of different number of turns are wound as shown in Figure 23.2.

The winding to which a.c voltage is impressed is called the *primary* of the transformer and the winding across which the load is connected is called the *secondary* of the transformer.

23.3 Ideal Transformer

To understand the working of a transformer it is always instructive, to begin with the concept of an *ideal* transformer with the following properties.

1. Primary and secondary windings has no resistance.

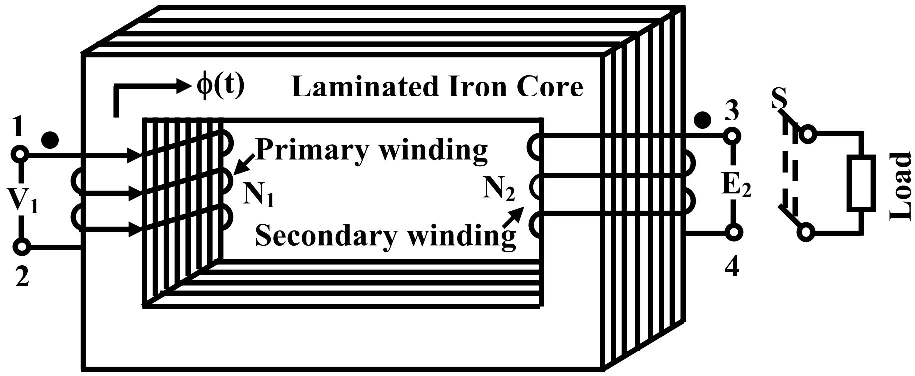


Figure 23.2: A typical transformer.

2. All the flux produced by the primary links the secondary winding i,e., there is no leakage flux.

3. Permeability μ_r of the core is infinitely large. In other words, to establish flux in the core vanishingly small (or zero) current is required.

4. Core loss comprising of *eddy current* and *hysteresis* losses are neglected.

23.3.1 Core flux gets fixed by voltage & frequency

The flux level B_{max} in the core of a given magnetic circuit gets fixed by the magnitude of the supply voltage and frequency. This important point has been discussed in the previous lecture 20. It was shown that:

$$B_{\max} = \frac{V}{\sqrt{2}\pi fAN} = \frac{1}{4.44AN}\frac{V}{f}$$

where, V is the applied voltage at frequency f, N is the number of turns of the coil and A is the cross sectional area of the core. For a given magnetic circuit A and N are constants, so B_{max} developed in core is decided by the ratio $\frac{V}{f}$. The peak value of the coil current I_{max}, drawn from the supply now gets decided by the B-H characteristics of the core material.

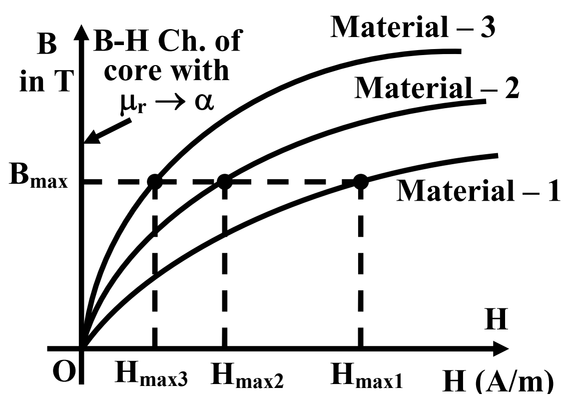


Figure 23.3: Estimating current drawn for different core materials.

To elaborate this, let us consider a magnetic circuit with *N* number of turns and core section area *A* with mean length *l*. Let material-3 be used to construct the core whose B-H characteristic shown in figure 23.3. Now the question is: if we apply a voltage *V* at frequency *f*, how much current will be drawn by the coil? We follow the following steps to arrive at the answer.

1. First calculate maximum flux density using $B_{\max} = \frac{1}{4.44AN}\frac{V}{f}$. Note that value of B_{max} is independent of the core material property.
2. Corresponding to this B_{max}, obtain the value of H_{max3} from the B-H characteristic of the material-3 (figure 23.3).
3. Now calculate the required value of the current using the relation $I_{\max 3} = \frac{H_{\max 3}l}{N}$.
4. The rms value of the exciting current with material-3 as the core, will be $I_3 = I_{\max 3}/\sqrt{2}$.

By following the above steps, one could also estimate the exciting currents (I_2 or I_3) drawn by the coil if the core material were replaced by material-2 or by material-3 with other things remaining same. Obviously current needed, to establish a flux of B_{max} is lowest for material-3. Finally note that if the core material is such that $\mu_r \to \infty$, the B-H characteristic of this ideal core material will be the B axis itself as shown by the thick line in figure 23.3 which means that for such an ideal core material current needed is practically zero to establish any B_{max} in the core.

23.3.2 Analysis of ideal transformer

Let us assume a sinusoidally varying voltage is impressed across the primary with secondary winding open circuited. Although the current drawn I_m will be practically zero, but its position will be 90° lagging with respect to the supply voltage. The flux produced will obviously be in phase with I_m. In other words the supply voltage will lead the flux phasor by 90°. Since flux is common for both the primary and secondary coils, it is customary to take flux phasor as the reference.

$$\text{Let, } \phi(t) = \phi_{max}\, sin\ \omega t$$
$$\text{then, } v_1 = V_{max} sin\left(\omega t + \frac{\pi}{2}\right) \qquad (23.1)$$

The time varying flux $\phi(t)$ will link both the primary and secondary turns inducing in voltages e_1 and e_2 respectively

$$\text{Instantaneous induced voltage in primary} = -N_1\frac{d\phi}{dt} = \omega N_1 \phi_{max} sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= 2\pi\, f\, N_1 \phi_{max} sin\left(\omega t - \frac{\pi}{2}\right) \quad (23.2)$$

Instantaneous induced voltage in secondary $= -N_2 \frac{d\phi}{dt} = \omega N_2 \phi_{max} sin\left(\omega t - \frac{\pi}{2}\right)$

$$= 2\pi\, f\, N_2 \phi_{max} sin\left(\omega t - \frac{\pi}{2}\right) \quad (23.3)$$

Magnitudes of the rms induced voltages will therefore be

$$E_1 = \sqrt{2}\pi\, f\, N_1\, \phi_{max} = 4.44\, f\, N_1 \phi_{max} \quad (23.4)$$

$$E_2 = \sqrt{2}\pi\, f\, N_2 \phi_{max} = 4.44\, f\, N_2 \phi_{max} \quad (23.5)$$

The time phase relationship between the applied voltage v_1 and e_1 and e_2 will be same. The 180° phase relationship obtained in the mathematical expressions of the two merely indicates that the induced voltage opposes the applied voltage as per *Lenz's law*. In other words if e_1 were allowed to act alone it would have delivered power in a direction opposite to that of v_1. By applying Kirchoff's law in the primary one can easily say that $V_1 = E_1$ as there is no other drop existing in this ideal transformer. Thus udder no load condition,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Where, V_1, V_2 are the terminal voltages and E_1, E_2 are the rms induced voltages. In convention 1, phasors $\overline{E}^1$ and $\overline{E}^2$ are drawn 180° out of phase with respect to $\overline{V}^1$ in order to convey the respective power flow directions of these two are opposite. The second convention results from the fact that the quantities $v_1(t)$, $e_1(t)$ and $e_2(t)$ vary in unison, then why not show them as co-phasal and keep remember the power flow business in one's mind.

23.3.3 No load phasor diagram

A transformer is said to be under no load condition when no load is connected across the secondary i.e., the switch S in figure 23.2 is kept opened and no current is carried by the secondary windings. The phasor diagram under no load condition can be drawn starting with $\overline{\phi}$ as the reference phasor as shown in figure 23.4.

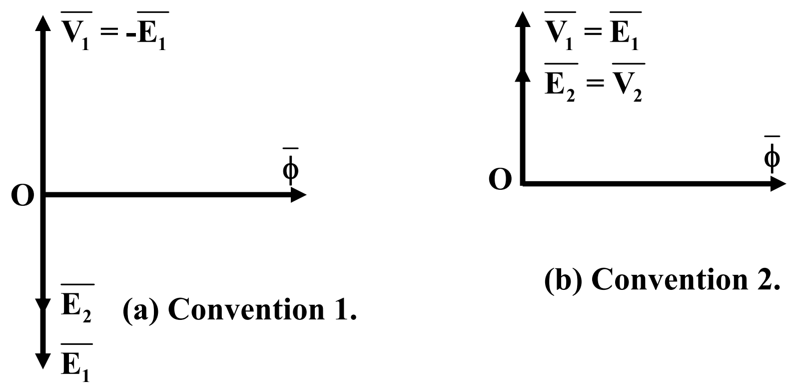


Figure 23.4: No load Phasor Diagram following two conventions.

In convention 1, phsors $\overline{E_1}$ and $\overline{E_2}$ are drawn 180° out of phase with respect to $\overline{V_1}$ in order to convey that the respective power flow directions of these two are opposite. The second convention results from the fact that the quantities $v_1(t)$, $e_1(t)$ and $e_2(t)$ vary in unison then why not show them as co-phasal and keep remember the power flow business in one's mind. Also remember vanishingly small magnetizing current is drawn from the supply creating the flux and in time phase with the flux.

23.4 Transformer under loaded condition

In this lesson we shall study the behavior of the transformer when loaded. A transformer gets loaded when we try to draw power from the secondary. In practice loading can be imposed on a transformer by connecting impedance across its secondary coil. It will be explained how the primary reacts when the secondary is loaded. It will be shown that any attempt to draw current/power from the secondary, is immediately responded by the primary winding by drawing extra current/power from the source. We shall also see that mmf balance will be maintained whenever both the windings carry currents. Together with the *mmf balance equation* and *voltage ratio* equation, invariance of Volt-Ampere (VA or KVA) irrespective of the sides will be established.

We have seen in the preceding section that the secondary winding becomes a seat of emf and ready to deliver power to a load if connected across it when primary is energized. Under no load condition power drawn is zero as current drawn is zero for ideal transformer. However when loaded, the secondary will deliver power to the load and same amount of power must be sucked in by the primary from the source in order to maintain *power balance*. We expect the primary current to flow now. Here we shall examine in somewhat detail the mechanism of drawing extra current by the primary when the secondary is loaded. For a fruitful discussion on it let us quickly review the *dot* convention in mutually coupled coils.

23.4.1 Dot convention

The primary of the transformer shown in figure 23.2 is energized from a.c source and potential of terminal 1 with respect to terminal 2 is $v_{12} = V_{max} sin\omega t$. Naturally polarity of 1 is sometimes +ve and some other time it is –ve. The *dot* convention helps us to determine the polarity of the induced voltage in the secondary coil marked with terminals 3 and 4. Suppose at some time *t* we find that terminal 1 is +ve and it is increasing with respect to terminal 2. At that time what should be the status of the induced voltage polarity in the secondary – whether terminal 3 is +ve or –ve? If possible let us assume terminal 3 is –ve and terminal 4 is positive. If that be current the secondary will try to deliver current to a load such that current comes out from terminal 4 and enters terminal 3. Secondary winding therefore, produces flux in the core in the same direction as that of the flux produced by the primary. So core flux gets strengthened in inducing more voltage. This is contrary to the dictate of Lenz's law which says that the polarity of the induced voltage in a coil should be such that it will try to oppose the cause for which it is due. Hence terminal 3 can not be –ve.

If terminal 3 is +ve then we find that secondary will drive current through the load leaving from terminal 3 and entering through terminal 4. Therefore flux produced by the secondary clearly opposes the primary flux fulfilling the condition set by Lenz's law. Thus when terminal 1 is +ve terminal 3 of the secondary too has to be positive. In mutually coupled coils dots are put at the appropriate terminals of the primary and secondary merely to indicative the status of polarities of the voltages. Dot terminals will have at any point of time identical polarities. In the transformer of figure 23.2 it is appropriate to put dot markings on terminal 1 of primary and terminal 3 of secondary. It is to be noted that if the *sense* of the windings are known (as in figure 23.2), then one can ascertain with confidence where to place the dot markings without doing any testing whatsoever. In practice however, only a pair of primary terminals and a pair of secondary terminals are available to the user and the sense of the winding can not be ascertained at all. In such cases the dots can be found out by doing some simple tests such as *polarity test* or *d.c kick test*.

If the transformer is loaded by closing the switch *S*, current will be delivered to the load from terminal 3 and back to 4. Since the secondary winding carries current it produces flux in the anti clock wise direction in the core and tries to reduce the original flux. However, KVL in the primary demands that core flux should remain constant no matter whether the transformer is loaded or not. Such a requirement can only be met if the primary draws a definite amount of extra current in order to nullify the effect of the mmf produced by the secondary. Let it be clearly understood that net mmf acting in the core is given by: mmf due to vanishingly small magnetizing current + mmf due to secondary current + mmf due to additional primary current. But the last two terms must add to zero in order to keep the flux constant and net mmf eventually be once again be due to vanishingly small magnetizing current. If I_2 is the magnitude of the secondary current and $I_2^{'}$ is the additional current drawn by the primary then following relation must hold good:

$$N_1 I'_2 = N_2 I_2$$

$$\text{or } I'_2 = \frac{N_2}{N_1} I_2$$

$$= \frac{I_2}{a}$$

$$\text{where, } a = \frac{N_1}{N_2} = \text{turns ratio} \qquad (23.6)$$

To draw the phasor diagram under load condition, let us assume the power factor angle of the load to be θ_2, lagging. Therefore the load current phasor $\overline{I_2}$, can be drawn lagging the secondary terminal voltage $\overline{E_2}$ by θ_2 as shown in the figure 23.5.

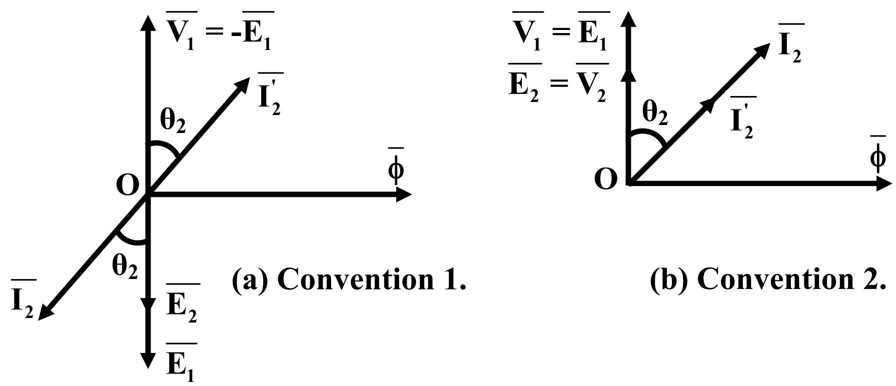


Figure 23.5: Phasor Diagram when transformer is loaded.

The reflected current magnitude can be calculated from the relation $I'_2 = \frac{I_2}{a}$ and is shown directed 180° out of phase with respect to $\overline{I_2}$ in convention 1 or in phase with $\overline{I_2}$ as per the convention 2. At this stage let it be suggested to follow one convention only and we select convention 2 for that purpose. Now,

$$\text{Volt-Ampere delivered to the load} = V_2 I_2$$

$$= E_2 I_2$$

$$= aE_1 \frac{I_1}{a}$$

$$= E_1 I_1 = V_1 I_1 = \text{Volt-Ampere drawn from the supply.}$$

Thus we note that for an ideal transformer the output VA is same as the input VA and also the power is drawn at the same power factor as that of the load.

23.4.2 Equivalent circuit of an ideal transformer

The equivalent circuit of a transformer can be drawn (i) showing both the sides along with parameters, (ii) referred to the primary side and (iii) referred to the secondary side.

In which ever way the equivalent circuit is drawn, it must represent the operation of the transformer correctly both under no load and load condition. Figure 23.6 shows the equivalent circuits of the transformer.

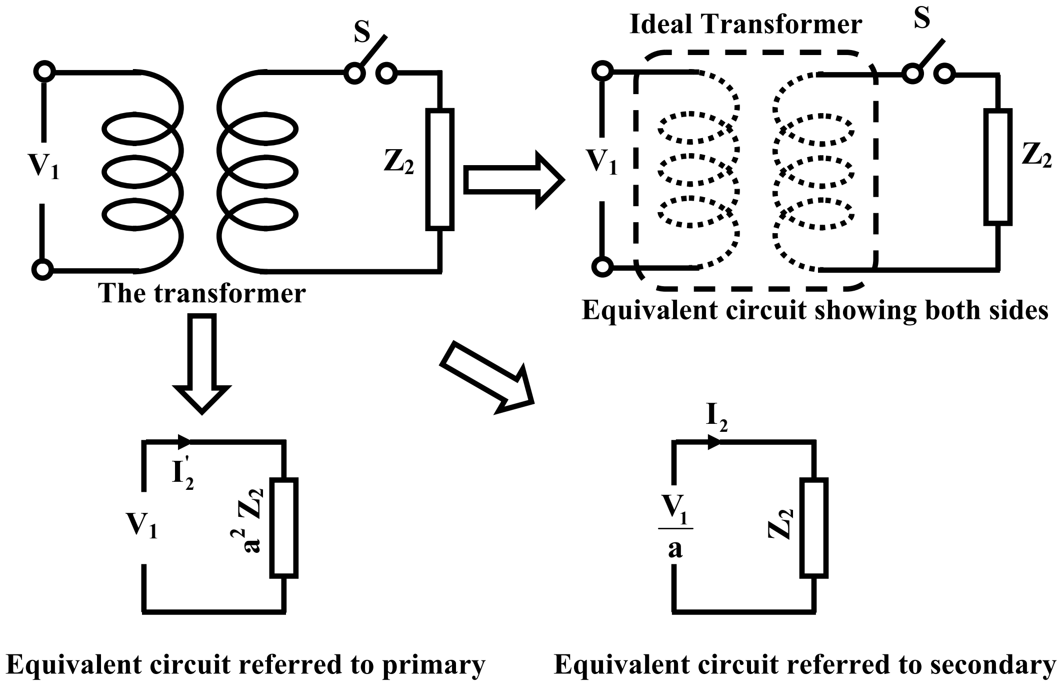


Figure 23.6: Equivalent circuits of an ideal transformer.

Think in terms of the supply. It supplies some current at some power factor when a load is connected in the secondary. If instead of the transformer, an impedance of value a^2Z_2 is connected across the supply, supply will behave identically. This corresponds to the equivalent circuit referred to the primary. Similarly from the load point of view, forgetting about the transformer, we may be interested to know what voltage source should be impressed across Z_2 such that same current is supplied to the load when the transformer was present. This corresponds to the equivalent circuit referred to the secondary of the transformer. When both the windings are shown in the equivalent circuit, they are shown with chain lines instead of continuous line. Why? This is because, when primary is energized and secondary is opened no current is drawn, however current is drawn when a load is present on the secondary side. Although supply two terminals are physically joined by the primary winding, the current drawn depends upon the load on the secondary side.

23.5 Tick the correct answer

1. An ideal transformer has two secondary coils with number of turns 100 and 150 respectively. The primary coil has 125 turns and supplied from 400 V, 50 Hz, single phase source. If the two secondary coils are connected in series, the possible voltages across the series combination will be:

(A) 833.5 V or 166.5 V (B) 833.5 V or 320 V
(C) 320 V or 800 V (D) 800 V or 166.5 V

2. A single phase, ideal transformer of voltage rating 200 V / 400 V, 50 Hz produces a flux density of 1.3 T when its LV side is energized from a 200 V, 50 Hz source. If the LV side is energized from a 180 V, 40 Hz source, the flux density in the core will become:

   (A) 0.68 T (B) 1.44 T (C) 1.62 T (D) 1.46 T

3. In the coil arrangement shown in Figure 23.7, A dot (•) marking is shown in the first coil. Where would be the corresponding dot (•) markings be placed on coils 2 and 3?

   (A) At terminal *P* of coil 2 and at terminal *R* of coil 3
   (B) At terminal *P* of coil 2 and at terminal *S* of coil 3
   (C) At terminal *Q* of coil 2 and at terminal *R* of coil 3
   (D) At terminal *Q* of coil 2 and at terminal *S* of coil 3

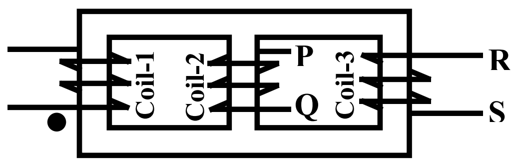
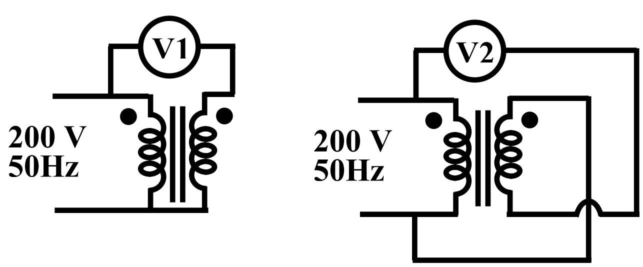


Figure 23.7:

4. A single phase ideal transformer is having a voltage rating 200 V / 100 V, 50 Hz. The HV and LV sides of the transformer are connected in two different ways with the help of voltmeters as depicted in figure 23.8 (a) and (b). If the HV side is energized with 200 V, 50 Hz source in both the cases, the readings of voltmeters V1 and V2 respectively will be:

   (A) 100 V and 300 V (B) 300 V and 100 V
   (C) 100 V or 0 V (D) 0 V or 300 V



Connection (a) **Connection (b)**

Figure 23.8:

5. Across the HV side of a single phase 200 V / 400 V, 50 Hz transformer, an impedance of $32 + j24\Omega$ is connected, with LV side supplied with rated voltage & frequency. The supply current and the impedance seen by the supply are respectively:

   (A) 20 A & $128 + j96\Omega$ (B) 20 A & $8 + j6\Omega$
   (C) 5 A & $8 + j6\Omega$ (D) 20 A & $16 + j12\Omega$

6. The rating of the primary winding a transformer, having 60 number of turns, is 250 V, 50 Hz. If the core cross section area is 144 cm^2 then the flux density in the core is:

   (A) 1 T (B) 1.6 T (C) 1.4 T (D) 1.5 T

23.6 Solve the following

1. In Figure 23.9, the *ideal transformer* has turns ratio 2:1. Draw the equivalent circuits referred to primary and referred to secondary. Calculate primary and secondary currents and the input power factor and the load power factor.

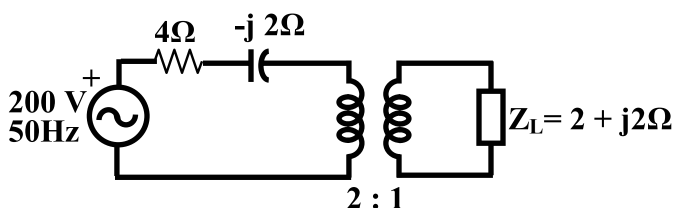


Figure 23.9: Basic scheme of protection.

2. In the Figure 23.10, a 4-winding transformer is shown along with number of turns of the windings. The first winding is energized with 200 V, 50 Hz supply. Across the 2nd winding a pure inductive reactance $X_L = 20\ \Omega$ is connected. Across the 3rd winding a pure resistance $R = 15\ \Omega$ and across the 4th winding a capacitive reactance of $X_C = 10\ \Omega$ are connected. Calculate the input current and the power factor at which it is drawn.

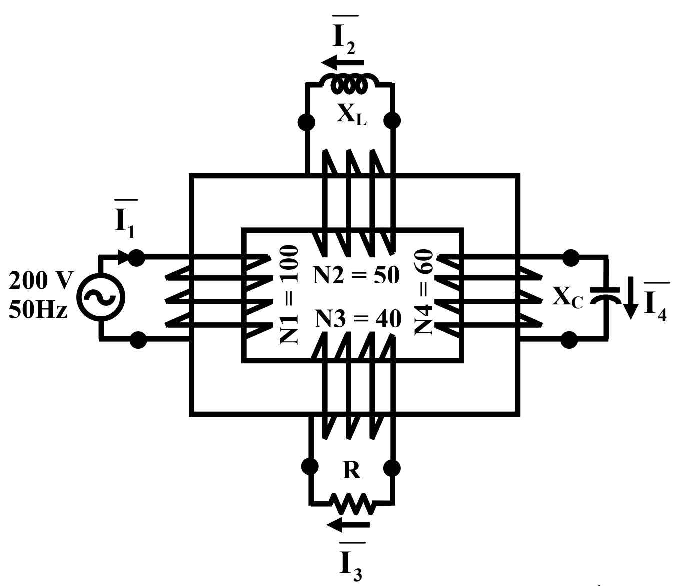


Figure 23.10:

3. In the circuit shown in Figure 23.11, T1, T2 and T3 are *ideal* transformers.

   a) Neglecting the impedance of the transmission lines, calculate the currents in primary and secondary windings of all the transformers. Reduce the circuit refer to the primary side of T1.
   b) For this part, assume the transmission line impedance in the section AB to be $\overline{Z}_{AB} = 1 + j3\Omega$. In this case calculate, what should be $\overline{V}_s$ for maintaining 450 V across the load $\overline{Z}_L = 60 + j80\Omega$. Also calculate the net impedance seen by $\overline{V}_s$.

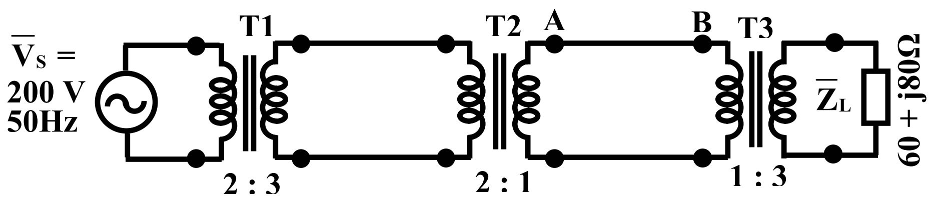


Figure 23.11:

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